

Reignition of Metallic A-C. Arcs in Air

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Synopsis.—Developments made in circuit breakers in the last two years have emphasized the necessity for obtaining experimental evidence of the current-voltage-time relationships that exist during the period when the a-c. arc between metallic electrodes passes through its cyclic current zero. Twenty-nine cathode ray oscillograms of these relationships are presented.

During the current zero period the arc electrode voltage is determined by the circuit constants and rises until the electrode voltage reaches the breakdown or reignition value, which is determined by the deionizing influences at work while the arc is extinguished. Altera-

tion of the circuit constants permits a variation in the rate of voltage rise with a consequent change in the reignition voltage.

Permanent extinguishment of the arc occurs when the gap breakdown voltage has risen, due to deionizing influences, to a value that cannot be reached by the electrode voltage controlled by circuit constants.

The action of a circuit breaker in extinguishing an arc is greatly influenced by the presence of adjacent load circuits and by the presence of distributed inductance and capacity in the connecting lines.

INTRODUCTION

RECENT work on the general subject of arc extinction as related to switch and circuit breaker action has emphasized the need for more accurate information concerning the conditions existing in an a-c. arc during the period of current zero. In particular, experimental evidence has been needed to show the rate at which the arc voltage rises toward a reignition value in the new circuit direction. This paper presents a study of the voltage and current relationships near the cyclic current zero in arcs between metallic electrodes, at moderate values of current and voltage, based on some two hundred cathode-ray oscillograms. The cathode-ray oscillograph is necessary inasmuch as the voltage may change at the rate of several hundred millions per second.

Records were taken with various circuit conditions, and with different electrode materials, shapes, and spacings. The electron beam in the oscillograph was initiated by a surge circuit controlled by a rotating contact synchronously driven from the same voltage source as that which supplied the arc. The most consistent results were obtained with an arc between copper plates 8 in. sq. and $\frac{3}{4}$ of an in. apart. This arrangement gives an arc of constant length and one which reaches its current zero at nearly the same part of each half cycle. For most of the work the power source was a transformer supplying 440 volts at 60 cycles. However, for certain circuit arrangements requiring high reignition voltages a 560-volt supply was used. The currents used were 25 amperes or less.

This investigation shows that when a small value of arc current is reached the current breaks sharply to substantially zero, where it remains until the discharge starts again with the opposite polarity. During this period the voltage across the arc electrodes rises according to the usual circuit equations until breakdown

occurs, yielding first a glow discharge in the arc gap and later an arc with the new polarity. Variation of the circuit constants offers the opportunity to alter the rate of the voltage rise. It is found that a voltage rising slowly must rise to a relatively large value before breakdown occurs. The curve of breakdown voltage against time has been discussed by Dr. Slepian from a theoretical basis.¹ This study presents the experimental evidence that permits this curve to be found and shows the modifications that must be made in it.

Moreover these oscillograms show the influence that nearby loads exert upon arc extinction in circuit breakers and emphasize the necessity for further study

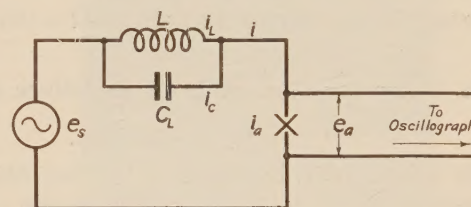


FIG. 1—CIRCUIT FOR UNSHUNTED ARC

of deionizing agents in the arc gap and of the influence of long lines with distributed constants upon arc extinction.

Arc Current—Voltage Relations

I. UNSHUNTED ARC

The arc current in the tests made was limited by an air core series reactor of sufficient size to make the current lag nearly 90 deg. behind the supply voltage. Fig. 1 shows the circuit composed of the transformer of voltage e_s connected through the reactor L , with its small (but not negligible) distributed capacitance C_L , to the arc. The arc voltage e_a was measured by the oscillograph. The time relationships existing between the supply voltage e_s , the arc voltage e_a and the arc current i_a are shown qualitatively in Fig. 2. Except

1. *Extinction of a Long A-C. Arc*, J. Slepian, A. I. E. E. TRANS., Vol. 49, 1930, p. 421.

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at instants near the current zero the current is practically a sine wave nearly 90 deg. behind the supply voltage. Throughout this part of the cycle the arc current i_a is practically the same as the current in the inductance, i_L .

Eventually the arc current reduces sinusoidally to a small value (order of 0.1 ampere) I_1 at which it suddenly fails, falling very sharply to practically zero, where it remains until the arc again is made with current in the opposite sense. In some cases this transition period to the arc in the new direction has lasted for 600 micro-seconds, which is 7 per cent of one-half cycle at 60 cycles. Throughout this period the supply voltage is practically constant ($e_s = E$).

The current i_L in the inductance cannot drop instantaneously and therefore surges into its own distributed capacitance. From $t = 0$ to $t = t_1$ the voltage e_a may be computed by the usual circuit equations, inasmuch as the arc current is practically zero. See Appendix A.

$$e_a = E - E_m \cos(\omega t + \theta) \quad (1)$$

where

$$\omega = \frac{1}{\sqrt{L C_L}} \quad E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C_L}\right)^2}$$

E = the maximum of the supply voltage.

E_1 = the arc voltage when the arc fails.

I_1 = the arc current when the arc fails.

e_a = the arc voltage during the period of $t = 0$ to $t = t_1$

$$\theta = \tan^{-1} \frac{I_1 / \omega C_L}{E - E_1} = \text{angle from arc failure to negative maximum.}$$

Particular note should be made of the fact that both the magnitude and the phase position of the negative voltage maximum is determined largely by the magnitude of the arc current when the arc fails. The frequency of the oscillation represented by this equation was of the order of 30,000 cycles per sec. in the circuits used. During this period from $t = 0$ to $t = t_1$ the current has changed from an extremely small negative value (much less than the 0.1 ampere arc-failure current) through zero at the zero of voltage to a small positive value. The current appears to change at approximately a uniform rate throughout this interval.

At time t_1 the voltage across the electrodes no longer rises. In fact it drops quickly to a voltage e_g , where it remains practically constant for a period of from zero to as long as 500 microseconds. The steady voltage e_g is indicative of the existence of a glow state (high-voltage, small current arc). The glow voltage is of the order of 350 volts. During the glow the current though still very small rises at a considerably greater rate than during the preceding period. Eventually the glow current rises to a value sufficient to maintain an arc proper (low-voltage, relatively high-current phenomenon). This change occurs at $t = t_2$. At this moment

the electrode voltage drops with great rapidity from the glow value of about 350 to the arc value of about 75, the latter depending on the length of the arc. Throughout the remainder of the half cycle the arc voltage remains constant, but the arc current exhibits its sine character.

Typical oscillograms illustrating this circuit condition for arcs between iron, aluminum, and copper electrodes respectively are shown in Figs. 3A, B, and C. In these three figures the starting glow voltages are 360, 310, and 340. The breakdown voltages preceding the glows are 380, 320, and 380 occurring in 24, 27, and 20 micro-seconds respectively. There appears to be far less difference between these kinds of electrodes, at least, than between various circuit conditions for only one electrode material.

It has been shown that the negative electrode voltage dip is due to the sharp break in the negative arc current and depends for its magnitude upon the value of the current at the moment it fails. It is possible for the

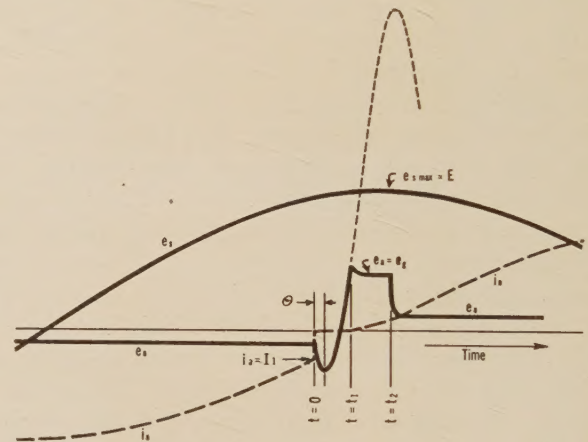


FIG. 2—UNSHUNTED ARC CURRENT AND VOLTAGE

current to fail earlier in the cycle at a larger negative value. In this case the negative electrode voltage may drop very sharply, in practically linear manner with time, to a sufficient value to produce a negative glow which lasts for a few microseconds and dies out. After this the electrode voltage follows the usual cosine curve given by equation (1). Fig. 3D illustrates the two types of negative voltage dip that may occur. This particular oscillogram is one of an 0.0096 μ f. capacity shunted across the arc, but similar negative dips with the unshunted arc have been observed frequently.

The sudden changes of the volt-ampere relationships in the electrical discharge that is passing at low-current values are best understood in terms of the volt-ampere characteristics of the arc and glow discharge in air at atmospheric pressure. Compton² gives the type of characteristic shown in Fig. 4A, based on the use of direct current over long time intervals. The oscillo-

2. *The Physical Nature of the Electric Arc*, Compton, A. I. E. E. TRANSACTIONS, 1927, p. 868.

grams presented in this paper clearly show the existence of both the arc and glow states at low arc current values, but they indicate that the shape of the volt-ampere characteristic is appreciably different from that shown in Fig. 4A.

In the a-c. arc, with conditions changing with great

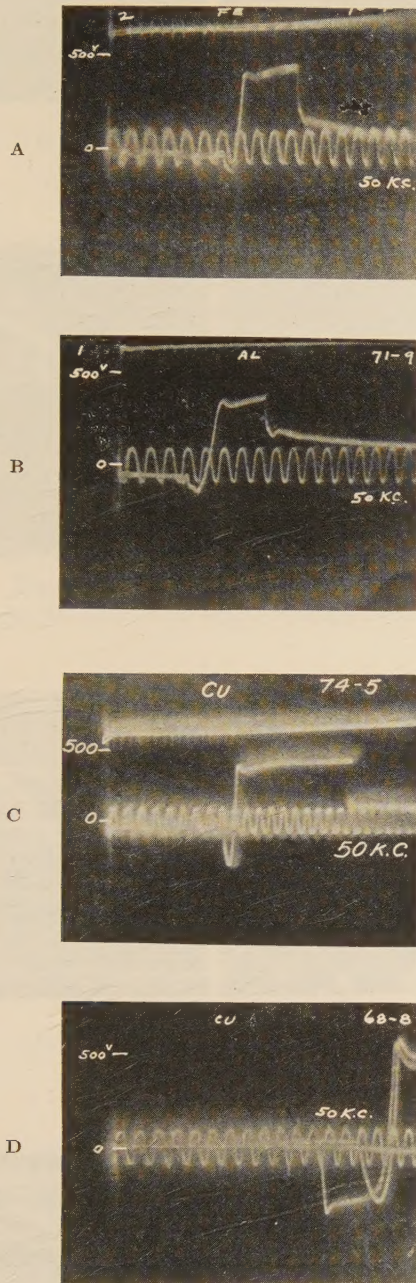
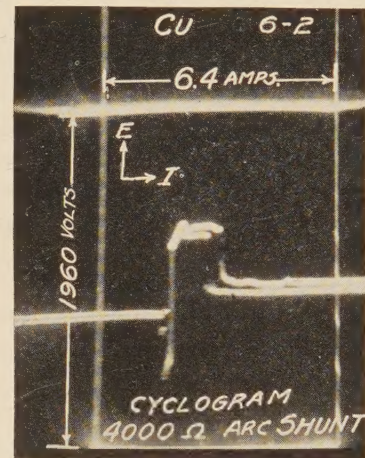
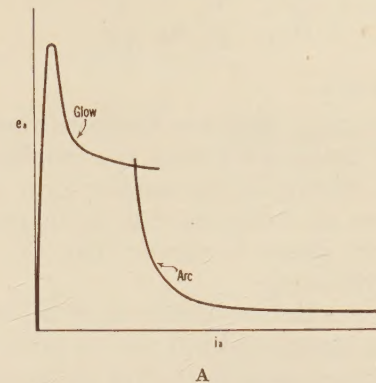


FIG. 3—VOLTAGE OSCILLOGRAMS FOR Fe, Al AND Cu, UNSHUNTED ARCS. 3D ILLUSTRATES NEGATIVE DIP (ROUND CURVE), AND NEGATIVE GLOW (PEAKED CURVE)

rapidity, the arc current may fail at larger or smaller values, the larger values producing a negative glow, the smaller values failing to do so. A positive arc voltage produces a very small current growing with the voltage until the glow voltage is reached, when the current rises at practically constant voltage. This condition

continues until the arc current has reached a value sufficient to maintain an arc proper, whereupon the voltage drops sharply (about 400 to 75) and the current returns to its sinusoidal variation at constant arc voltage. Fig. 4B is a cyclogram of arc voltage against arc current illustrating this point. This cyclogram was taken with a 4,000 ohms shunt across the arc, but it is representative of the unshunted arc case. It should be noted in the cyclogram that the curve has a different shape for the two polarities in the alternating current case, while an extension of Fig. 4A to include negative



B

FIG. 4—VOLTAGE CURRENT CYCLOGRAMS

- A. Direct current and long time (Compton)
- B. A-c. short time oscillogram

currents for the direct current situation would be a perfect reflection of the curve as shown but with reversed polarity.

II. RESISTANCE SHUNTED ARC

The circuit used for the resistance shunted arc is shown in Fig. 7A. The electrode voltage during the arc current zero for a resistance shunted arc must be written in two forms, depending upon whether the resistance R is greater or less than the critical resistance

R_c , which is equal to $\frac{1}{2} \sqrt{\frac{L}{C_L}}$. See Appendix B.

For resistances larger than the critical resistance the electrode voltage becomes

$$e_a = E - e^{-\frac{1}{2RC_L}t} \left[(E - E_1) \cos \omega t + \frac{1}{\omega C_L} \left(-I_1 + \frac{E - E_1}{2R} \sin \omega t \right) \right] \quad (2)$$

$$\omega = \sqrt{\frac{1}{LC_L} - \frac{1}{4R^2C_L^2}}$$

$$R > \frac{1}{2} \sqrt{\frac{L}{C_L}}$$

See Appendix B-I.

Figs. 5A, B and C illustrate this condition. For the first two of these cases with an infinite resistance equation (2) reduces to the simpler form of equation (1). Equation (2) holds for Fig. 5C inasmuch as the 10,000-ohm arc shunt is greater than the critical resistance of 6,125 ohms.

The circuit conditions back of all of the oscillograms of Fig. 5 were identical except for the shunt resistance, and the maximum value of the line voltage varied only slightly from 975. The series reactor possessed an inductance L of 0.066 henrys and a distributed capacity C_L of 440 μf . This distributed capacity may easily be calculated by applying equation (1) to one of the curves of Fig. 5B. From the magnitude and phase position of the negative dip maximum ω may be computed and hence C_L found. The arc was struck between the centers of two 8 in. by 8 in. parallel copper plates spaced $\frac{3}{4}$ of an in. apart. The arc was semi-enclosed to prevent effects from stray air currents.

For resistances smaller than the critical resistance the electrode voltage takes the form

$$e_a = E - e^{-\frac{1}{2RC_L}t} \left[(E - E_1) \cosh \beta t + \frac{-I_1 + \frac{E - E_1}{2R}}{\beta C_L} \sinh \beta t \right] \quad (3)$$

$$\beta = \sqrt{\frac{1}{4R^2C_L^2} - \frac{1}{LC_L}}$$

$$R < \frac{1}{2} \sqrt{\frac{L}{C_L}}$$

See Appendix B-II.

Figs. 5D, E, F, G and H for shunts of 4,000, 2,000, 1,200, 800 and 500 ohms respectively may be represented by equation (3).

For purposes of computation equation (4) may be used in place of equation (3) from which it has been reformed.

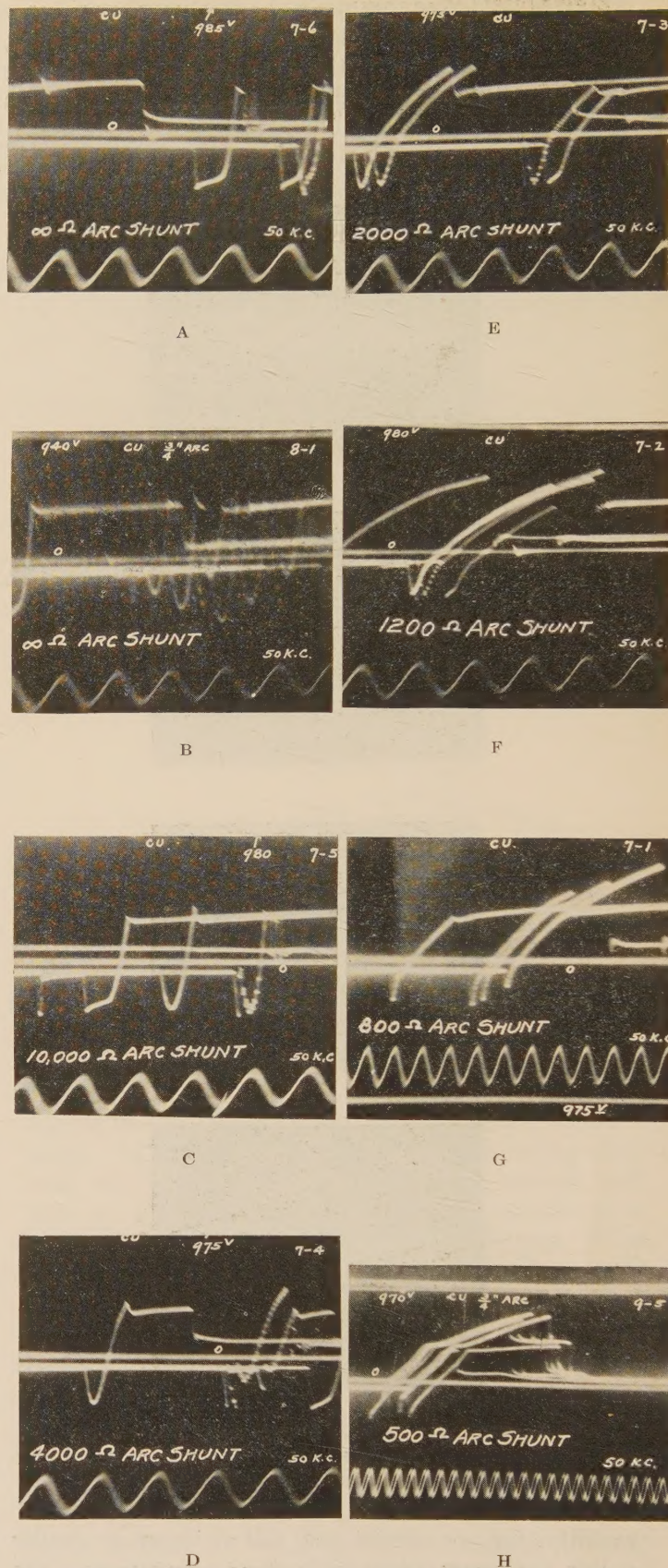


FIG. 5—VOLTAGE OSCILLOGRAMS FOR RESISTANCE SHUNTED ARC. 25-AMPERE, 660-VOLT, $\frac{3}{4}$ -IN. ARC BETWEEN FLAT COPPER ELECTRODES

$$e_a = E - \frac{1}{1 - \Delta_1} \left\{ e^{-\frac{Rt}{L}(1 + \Delta_2)} \left[(E - E_1) \left(1 - \frac{\Delta_1}{2} \right) - I_1 R \right] - e^{-\frac{t}{RC_1} \left(1 - \frac{\Delta_1}{2} \right)} \left[(E - E_1) \frac{\Delta_1}{2} - I_1 R \right] \right\} \quad (4)$$

where

$$\alpha = \frac{4 R^2 C_L}{L}$$

$$\Delta_1 = 1 - 2 R C_L \beta = 1 - \sqrt{1 - \alpha} = \frac{\alpha}{2} + \frac{\alpha^2}{8} + \frac{\alpha^3}{16} + \dots$$

$$\Delta_3 = \frac{2 \Delta_1}{\alpha} - 1 = \frac{\alpha}{4} + \frac{\alpha^2}{8} + \dots$$

As t increases the second term in the large bracket becomes very small, and when $t > \frac{\log_e 10}{\beta}$ the error

introduced by its omission is less than one per cent.

The 500-ohm shunt of Fig. 5H is the lowest resistance at which it was possible to operate the arc. At and near this border line resistance the electrode voltage shows a peculiar hump at approximately the glow voltage. Sometimes the electrode space breaks into the glow at

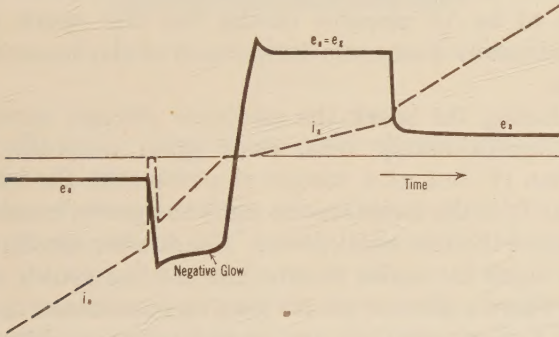


FIG. 6—VOLTAGE AND CURRENT FOR RESISTANCE SHUNTED Cu ARC WITH NEGATIVE GLOW

this stage and at other times the voltage rises to a higher breakdown point, then drops to glow or arc value.

The arc current curve takes the general form shown in Fig. 2 if the electrode voltage goes through its round negative dip. However, if the sharp voltage drop leading to a negative glow occurs (Fig. 6), the current drops to zero simultaneously but at once rises to a value sufficient to sustain a negative glow, after which it drops gradually throughout the existence of the negative glow, reaching zero at the end of the glow period. From this time on until the positive breakdown voltage is reached the current is extremely small and the voltage equations given hold.

The reactor current can be obtained by mechanical

integration from the electrode voltage curve, provided the reactor distributed capacity is relatively small.

$$E - e_a = L \frac{di}{dt}, \text{ so that}$$

$$i = \int (E - e_a) dt + K$$

Subtracting the current through the shunt resistance gives the arc current.

$$i_a = i - i_R = \int (E - e_a) dt + K - \frac{e_a}{R} \quad (5)$$

The effect of the arbitrary constant may be eliminated

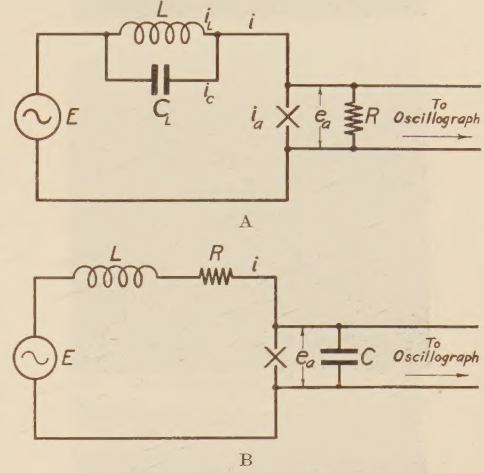


FIG. 7—CIRCUITS FOR RESISTANCE AND CAPACITY SHUNTED ARCS

- A Arc shunted by resistance
- B Arc shunted by capacity

by choosing the arc current as being zero when the arc voltage is zero.

III. CAPACITY SHUNTED ARC

The circuit used for the capacity shunted arc is shown in Fig. 7B. The small distributed capacity of the series reactor C_L may be neglected in comparison with the capacity across the arc C , inasmuch as the latter is far the greater.

The electrode voltage is given by equation (6), for

the case of $\frac{R^2}{4 L^2} < \frac{1}{L C}$. See Appendix C.

$$e_a = E + e^{-\frac{R}{2L} t} \left[- (E - E_1) \cos \omega t + \frac{I_1 - \frac{R C}{2 L} (E - E_1)}{\omega C} \sin \omega t \right] \quad (6)$$

$$\omega = \sqrt{\frac{1}{L C} - \frac{R^2}{4 L^2}} \quad \frac{R^2}{4 L^2} < \frac{1}{L C}$$

The studies presented in this paper involved a series resistance so low that it might be neglected. Making $R = 0$ equation (6) reduces to the form shown in equation (7), which is identical with equation (1)

$$\theta = \tan^{-1} \frac{I_1}{\omega C (E - E_1)} \quad (7)$$

$$E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C} \right)^2}$$

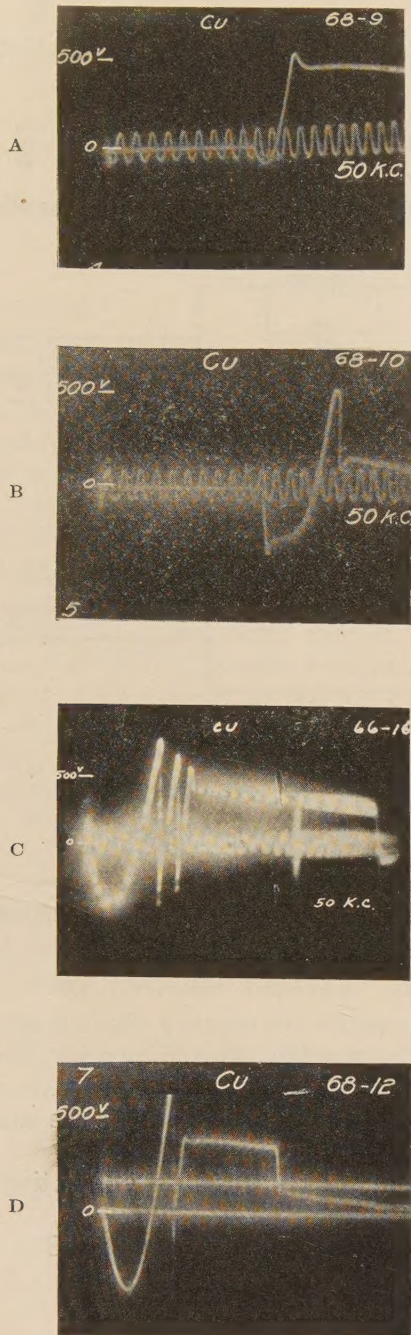


FIG. 8—VOLTAGE OSCILLOGRAMS FOR CAPACITY SHUNTED Cu ARC

providing the capacity of the coil is now replaced with that placed across the arc.

$$e_a = E - E_m \cos(\omega t + \theta)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

The electrode voltage, with capacity shunt, may exhibit the rounding negative dip or the sharp drop that leads to the negative glow, as shown in Figs. 8A and B. In these figures is shown also the possibility of having either a glow or an arc state immediately following the gap breakdown. The general character of these curves is similar to those for resistance shunted arcs. In the latter however the positive glow state almost universally appears.

All four electrode voltages shown in Fig. 8 were obtained by using a capacity of $0.0125 \mu\text{f}$. Figs. 8C and D illustrate a state that exists only with a relatively large capacity. In these figures it is seen that the voltage following the initial breakdown may break to a negative value or may develop into a sawtooth wave. Since the electrode and the capacity are shunted, the extremely rapid drop in voltage indicates a short time discharge of the condenser, or spark through the arc space. The negative voltage overshoot may be due to the small inductance of the condenser arc path or to some internal arc ionic condition. In the case of Fig. 12D, where the timing is stretched out sufficiently to permit measurement, the average arc current was found to be 13 amperes during the first spark and approximately 8 amperes during each of the succeeding sparks.

Following the spark the electrode voltage, starting from approximately zero, rises again according to equation (7) but at a steeper slope because the initial current I_1 in the series reactor path has grown considerably since the first breakdown. Spark after spark may follow until the series reactor current has grown to a point where a glow or an arc may be maintained in the gap. A sharp crackling and hissing noise accompanies the operation of this type of arc.

The oscillograms shown in Figs. 9A and B give the arc current and voltage for identical conditions, except that it was necessary to introduce a small coil into the arc path in order to measure the current. The capacity was $0.0125 \mu\text{f}$. The oscillograms were taken only a few seconds apart. The current picture shows clearly the initial break in the current, the prolonged current zero period and the spark that occurs whenever the electrode voltage drops suddenly. The current curve has been offset from the zero line. The arc current curve has approximately the same slope as the series reactor current for the condition of the arc short-circuited, as it should.

Figs. 10A and B are oscillograms of a cyclogram and an arc voltage taken a few seconds apart under the same conditions as those of the preceding figure. The

new point to be observed is the loop on the cyclogram which represents the current-voltage relation during the transition from the glow to the arc. Probably the loop should be pointed in the current direction; the rounding may be partially due to the inductance of the small coil inserted in the arc path for measurement purposes. The records on Fig. 11A, taken at intervals of a few seconds, show the current and voltage with the same capacity (0.0125 μ f.) when a sawtooth occurs.

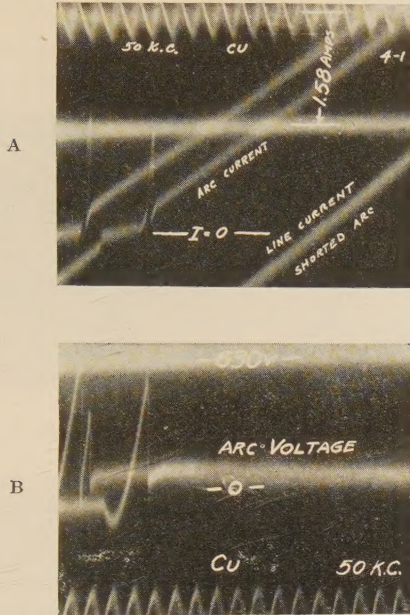


FIG. 9—CURRENT AND VOLTAGE OSCILLOGRAMS FOR 0.0125 μ f SHUNT

Fig. 11B is the complex type of cyclogram that may be expected in this case.

The effects of varying the capacity shunted across the arc are shown in Fig. 12. Below 0.01 μ f. the gap goes through the glow to reach the arc stage, but for larger capacities the sawtooth replaces the glow interval and the voltage changes require longer and longer periods. For the larger capacities the negative voltage dip is missing, not because the current continues until

I_1 is zero but because the term $\frac{I_1}{\omega C}$ in equation (7) becomes negligibly small compared to $E - E_1$.

ARC-FAILURE CURRENT VALUES

A knowledge of the numerical values of arc-failure currents is desirable. Now as soon as the arc stops momentarily at the end of a half-cycle the voltage recovery curve starts sharply downwards (increasing negative values) with a slope given by

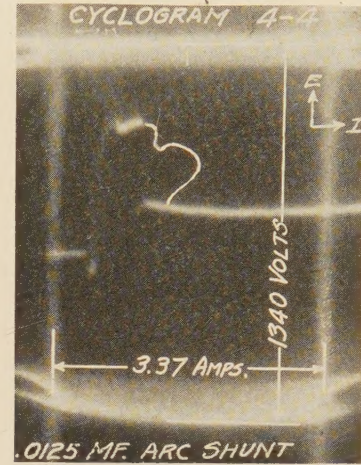
$$\left(\frac{d e_a}{d t} \right)_{t=0} = \frac{I_1}{C} \quad (8)$$

as shown in the appendixes.

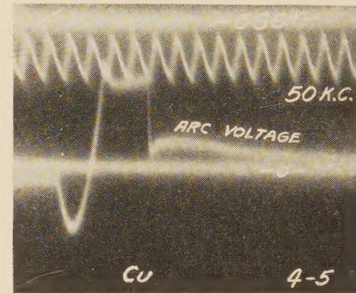
Theoretically this provides a measure of I_1 , which is

the arc-failure current for any particular record. Practically the slope is changing with considerable rapidity at this moment, so that it is impossible to obtain an accurate measure of the current by this means.

The trace ordinarily continues downward to a negative maximum which provides a good measure of the arc-failure current. Equation (9), obtained by equating the derivative of equation (4) to zero and rearranging, shows the relation between I_1 and the extreme of the negative dip, for the logarithmic case with a resistance shunt.



A



B

FIG. 10—CYCLOGRAM AND VOLTAGE FOR 0.0125 μ f SHUNT

$$I_1 = \frac{1 - \frac{\Delta_1}{2}}{R} \left[(E - E_1) - (E - e_m) T^{\frac{\Delta_2}{2}} \right] \quad (9)$$

where

$$T = e^{2\omega t_m} = \frac{\frac{2 I_1 R}{\Delta_1} - (E - E_1)}{\left(\frac{I_1 R}{1 - \frac{\Delta_1}{2}} \right) - (E - E_1)} \quad (10)$$

e_m = negative maximum voltage.

t_m = time at which it occurs.

$$\Delta_2 = \frac{1}{2 R C_L \beta} - 1 = \frac{1}{\sqrt{1 - \alpha}} - 1$$

$$= \frac{\alpha}{2} + \frac{3\alpha^2}{8} + \frac{5\alpha^3}{16} + \dots$$

Δ_1 , β and α have the meanings previously assigned.

$$R < \frac{1}{2} \sqrt{L/C_L}$$

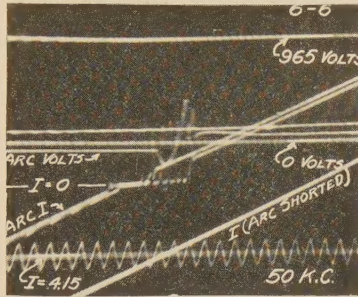
Equation (10) can readily be put into a form giving a direct solution for I_1 in terms of t_m , but in the records

$$I_1 = \frac{1}{R} (e_m - E_1) \quad (11)$$

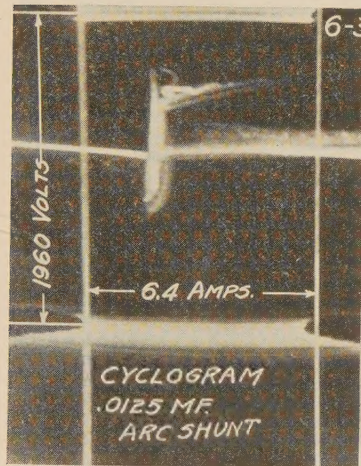
For the harmonic case with a resistance shunt equation (12), obtained in a manner similar to equation (9), provides the best approach to I_1 .

$$I_1 = -\sqrt{\frac{C_L}{L} \left[(E - e_m)^2 e^{\frac{t_m}{RC_L}} - (E - E_1)^2 \left(1 - \frac{1}{\alpha}\right) \right]}$$

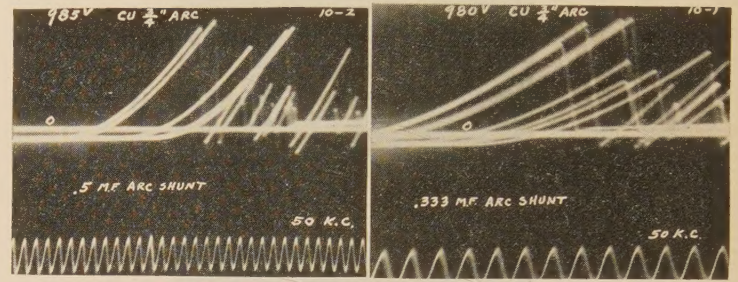
$$+ \frac{E - E_1}{2R} \quad (12)$$



A

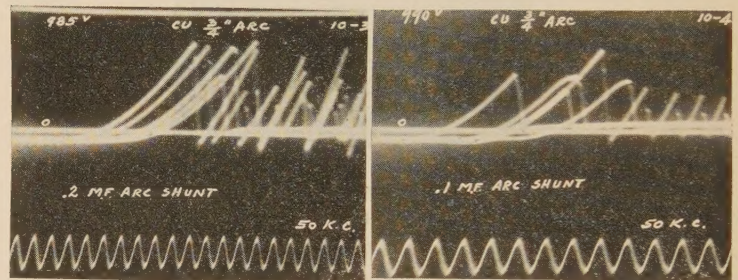


B



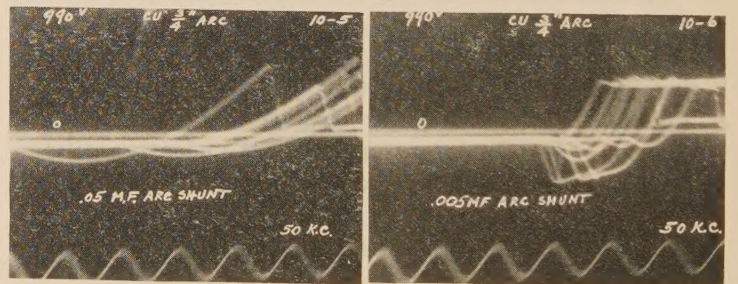
A

D



B

E



C

F

FIG. 11—CYCLOGRAM, CURRENT AND VOLTAGE FOR 0.0125 μf SHUNT

obtained t_m could not be accurately measured. It is easy, however, to measure e_m closely. I_1 is then obtainable by a trial and error solution of equation (9), in which $T^{\frac{\Delta_2}{2}}$ is only slightly sensitive to changes in I_1 .

The greatest value of $T^{\frac{\Delta_2}{2}}$ for all of the records studied was about 1.2, obtained in one of the 4,000-ohm cases; its minimum was of course unity.

For very small values of α , such as occur in the present study when R is 500 ohms, Δ_1 and Δ_2 can be neglected entirely, giving the still simpler form of equation (11).

FIG. 12—VOLTAGE OSCILLOGRAMS FOR CAPACITY SHUNTED ARC. 25-AMPERE, 660-VOLT, $\frac{3}{4}$ -IN. ARC BETWEEN FLAT COPPER ELECTRODES

where

$$\frac{t_m}{2 R C_L} = \frac{I_1}{I_1 - \frac{(E - E_1) \alpha}{2 R}}$$

$$R > \frac{1}{2} \sqrt{\frac{L}{C_L}}$$

As in the preceding case, a trial solution is required if t_m cannot be accurately measured.

For the capacity shunted arc, becomes E_m in equation (1), which is very readily solved for I_1 .

Of course if the arc-failure current is large enough so that the negative maximum that it would naturally reach exceeds the reignition voltage, a negative glow forms, and the normal extreme indicated by the circuit equations is never reached. This makes impossible an accurate determination of the arc-failure currents

large values. A low resistance is favorable toward arc failure at a relatively high value of arc current. A large capacity seems to have a similar tendency.

The maximum arc-failure current evaluated, considering all the observations with resistance and with capacitance in parallel with the arc, was less than 0.6 ampere. Some that resulted in a negative glow may have been slightly greater. The minimum determined was about 0.03 ampere. Values less than 0.05 ampere were very unusual.

ARC LEAKAGE WITH RESISTANCE SHUNTS

A peculiar behavior of considerable interest is exhibited most clearly in Fig. 5G. There is a sudden change in the slope of the voltage rise curve just before it reaches zero on the return from the negative peak. This is quite pronounced in the 1,200-ohm and 800-ohm records, and actually exists to a greater or less extent in all of the logarithmic cases except that for 4,000 ohms. In order to analyze it carefully the extreme left record of Fig. 5E has been reproduced as the solid line in Fig. 14. The lower dotted line represents the theoretical curve resulting from equation (3) that should be expected from an arc-failure current sufficient to give the negative voltage peak that does occur. The oscillographic record follows the theoretical curve very closely until shortly after the negative peak is reached, then for a short time the potential between the electrodes decreases much more rapidly than it would according to circuit theory. This is indicated by the departure of the voltage trace from the lower line.

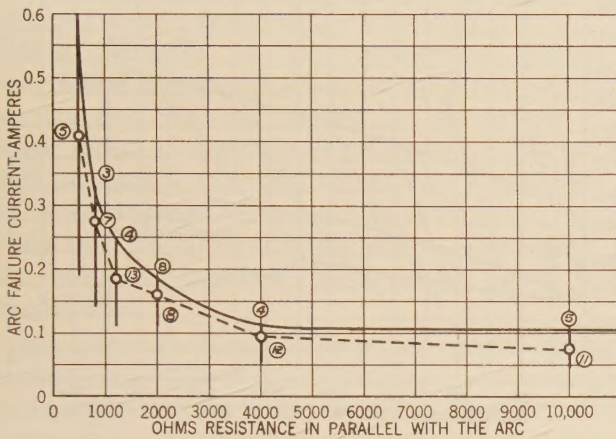


FIG. 13—ARC-FAILURE CURRENTS FOR RESISTANCE SHUNTS

for such cases; all that can be said is that they exceed a certain value.

Complete calculations have been made of the arc-failure currents from those traces that did not indicate a negative glow, the determinations being based on the extreme negative dip of the voltage recovery curve. The values obtained vary widely, but nevertheless exhibit a consistent trend in relation to shunt resistance values used. This is illustrated in Fig. 13, in which a heavy vertical line extends from the extreme lower to the upper value of arc-failure current observed with each resistance condition studied. The solid curve marking the upper ends of these lines merely indicates the boundary between currents that will and those that will not produce a negative glow.

Dotted lines join points that have been plotted to indicate the average of those arc-failure currents that did not produce a negative glow. Of course the omission of the glow cases prevents the averages from being of great significance, since every omission indicates an arc failure at a current near to or above the maximum of those contributing to the average. There is nevertheless a sufficiently definite trend to make the plot of interest. The figure adjacent to each point is the number of records contributing to the average, and at the top of each heavy vertical line is another figure which stands for the number of records made indicating a negative glow.

Fig. 13 indicates that the resistance acts in some way to affect the most likely value of arc-failure current, and to change the likelihood of very small or very

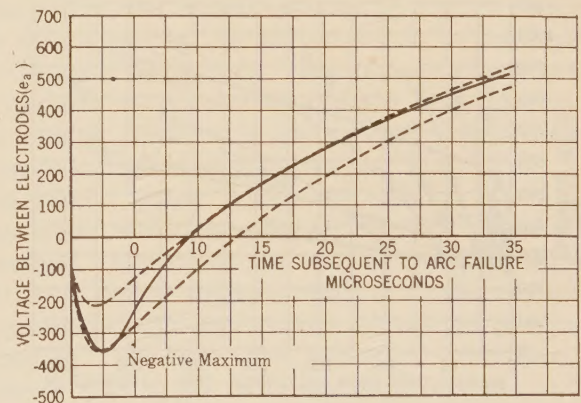


FIG. 14—ANALYSIS OF VOLTAGE CURVE FROM FIG. 5E TO SHOW EFFECT OF THE ARC-LEAKAGE CURRENT

As soon as zero voltage is reached it again follows a theoretical curve closely until reignition. The new curve has the same decrement and final asymptote as the original one, but differs in reference to initial conditions.

This indicates that for a few microseconds after the negative peak an appreciable "leak" of current takes place between the electrodes, without, however, resulting in a negative glow at its characteristic voltage. No such behavior as is illustrated in Fig. 14 appeared to take place in the capacity shunt observations.

REIGNITION VOLTAGE AS RELATED TO CIRCUIT CONSTANTS

One of the primary objectives of this investigation has been the determination of the nature of the relationship that exists between the value of voltage required for reignition and the time taken to reach that voltage. A program to meet this requirement necessitated modifying the circuit constants in such a way as to cause the voltage to rise at widely different rates. This was done by the use, first of resistance, then capacity, in parallel with the arc. The results of the resistance tests appear in Fig. 5, and those with the capacity in Fig. 12.

In making the tests for Figs. 5 and 12 considerable pains were taken to maintain the arc length, current, and power consumption constant. In spite of this quite a variation in reignition voltage appeared, especially in the case of slowly rising voltages. The results of all the tests that were made on the $\frac{3}{4}$ -in. 25-ampere 660-volt circuit arc are combined in Figs. 15 and 16. Each point on these graphs represents a reignition voltage, plotted against time measured from the instant at which the arc current failed.

Referring back to Fig. 14, it is seen that after zero voltage the recovery curve with a resistance shunt follows a theoretical trace dependent on the logarithmic decrement determined by circuit constants, on the asymptote determined by line voltage, and on the time at which zero voltage occurs. The dotted lines in Fig. 15 are such curves calculated from equation (4).

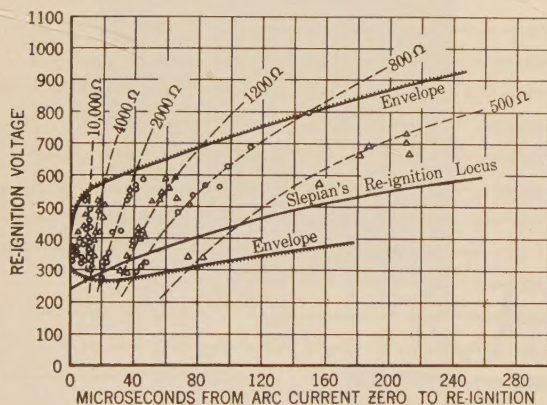


FIG. 15—REIGNITION VOLTAGE—TIME VALUES FOR RESISTANCE SHUNTS

25-ampere, 660-volt, $\frac{3}{4}$ -in. arc between flat copper electrodes

In placing them the zero voltage time was selected so to make the curve lie as close as possible to all of the observed reignition points.

In Fig. 16, representing the capacity shunt condition, the dotted lines that appear for each value of capacity used are theoretical voltage rise curves calculated from equation (7). The values of arc failure currents used in the calculations were chosen to produce curves best fitting the observed reignition points. These currents may be regarded as representative arc-failure currents

for each set of circuit conditions. The occasional points at some distance from the curves merely indicate arc-current failure at some value considerably different from the representative value chosen for the curve. For the 0.05, 0.2, and 0.5 μ f. capacity conditions the most representative arc-failure currents were so small, relative to other terms in the equation, that they had a negligible effect on the positions of the curves.

A small group of points to the extreme left of both figures, between 330 and 380 volts, and between one

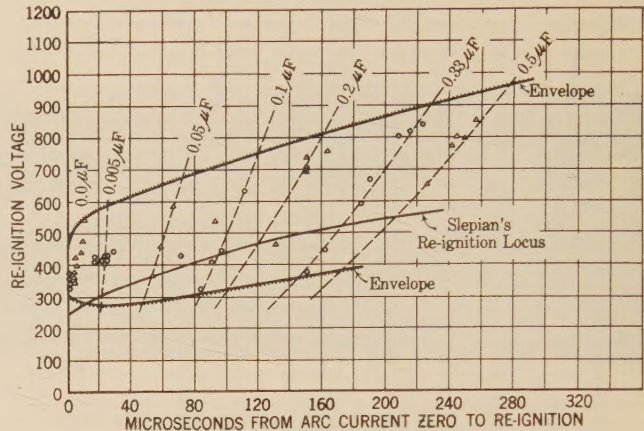


FIG. 16—REIGNITION VOLTAGE—TIME VALUES FOR CAPACITY SHUNTS

25-ampere, 660-volt, $\frac{3}{4}$ -in. arc between flat copper electrodes

and two microseconds, is taken from ignition points of the negative glow. These fit very well into the general appearance of the rest of the diagram; this tends to discount any supposition that would make the change in polarity of importance in determining the short-time re-striking voltage.

Closely related to the requirement of a generally higher re-striking voltage for smaller resistances or larger capacities there developed a consistently increasing difficulty in maintaining the arc for any great length of time. After only a few seconds of operation with small resistances or large capacities, and at some apparently accidentally determined half-cycle, the required re-striking voltage continued indefinitely to exceed the recovering voltage, hence the arc ceased.

The upper and lower envelope curves that appear on Figs. 15 and 16 provide a demarcation of the zone within which reignition occurred. It is interesting to note that the reignition locus obtained by Slepian using an indirect method is in general similar in shape to the upper envelope, but starts at a short-time value of 235 volts and is correspondingly low in value throughout its length. This locus is plotted on Figs. 15 and 16 for comparison.

The general features of these locus figures may be summed up as follows:

(1) For a very short time delay, the reignition voltage is quite definite in value, and is about the same as the glow voltage, but perhaps a trifle higher.

(2) As the voltage rise is delayed, the reignition voltage value varies between wide limits, the maximum rising considerably above the glow voltage, the minimum not changing greatly.

(3) There is no very marked difference between the reignition voltage required for a given time delay produced by resistance and that produced by capacity. The envelope curves in the two diagrams are identical.

It seems difficult to account for the great variation in late reignition voltages in terms of the various de-ionization processes that have been supposed to exist; especially is this true for the occasional very low re-striking voltages. In the experiments leading to Figs. 15 and 16 the ends of the arc were not stationary; they traveled, relatively slowly, on the surfaces of the copper plates that served as electrodes. The motion, while not rapid, was certainly sufficient to expose the arc to operation on surfaces differing from one moment to the next due to local irregularities. It may be that the variations observed resulted from these surface irregularities.

In a glow discharge the electron stream from the cathode leaves the body of the metal as a result of a combination of several causes, among them being bombardment by positive ions and the existence of a potential gradient due to a space-charge adjacent to the metal surface. The severity of the ion bombardment is of course closely related to the potential gradient at the cathode and to the space-charge. It seems reasonable to suppose that when some critical gradient at the surface is reached, electrons are suddenly released freely enough to support a glow discharge. In this case the reignition voltage might be expected to depend somewhat on surface irregularities, surface temperature, and space-charge distribution in the arc path, all of these being variable elements.

APPLICATION TO POWER SYSTEMS

The facts that have been presented are of interest in a consideration of conditions under which circuit breakers on electric power systems must operate. Manufacturers have observed that circuit breaker test installations in which reactance coils are used to limit short-circuit current impose an actual circuit breaker duty more severe than that resulting from the same current interruption requirement in normal service on a power system. The difference is attributed to the introduction of a considerable capacity in parallel with the arc by the lines and apparatus that limit the short-circuit current, and to the production of a condition roughly equivalent to a parallel resistance by the presence of load circuits. Both situations delay the voltage rise toward reignition, thereby permitting a longer time for recovery of dielectric strength in the arc region. This of course favors an early extinguishing of the arc.

A single-phase short circuit is usually interrupted by an oil switch in which there are two breaks in series in

each of the two line wires involved. There will then be four arcs in series in which reignition must occur if the arc is to persist after the end of any given half-cycle. Due to the variability of the reignition voltage, it is not to be supposed that all four arcs will reach re-striking voltage at exactly the same instant, but as soon as one breaks down the others will follow immediately due to the increased voltage imposed on them. If the electrostatic fields in the circuit breaker are such as to produce an approximately even distribution of the total voltage between the four breaks, the over-all reignition voltage will be approximately four times that for any one arc.

In oil circuit breakers the arc is drawn in oil. After a very short time it is playing in a bubble of gas formed by the decomposition of oil. The reignition and glow voltages cannot be expected to be the same in these gases as in air, but they should be of the same order of magnitude. The total reignition voltage can then be represented in the manner of Figs. 15 and 16 except with voltages in general about four times as great.

The calculation of the voltage rise curves for a transmission circuit is complicated by the necessity of considering distributed capacity and inductance in their relation to the time required for the propagation of an electrical impulse along a line. Thus it requires about 50 microseconds for an impulse to traverse 10 miles of line; therefore it would seem that only the capacity of that part of the line less than 10 miles from the circuit breaker can effect the voltage across the arc during the first 50 microseconds of voltage rise. Furthermore, the effect of the distributed capacity of a given section of line will depend on the inductance to be traversed by the charge in reaching it. The authors plan to present an analysis of this situation in the near future. For the present, however, a few general observations can be made as to what is likely to occur.

In general, power system voltages are much higher than those used in securing the results that have been described. Equations (1), (2), (3) and (4) all show that these higher voltages result in a tendency toward a correspondingly greater rate of voltage rise toward reignition. However, the values of inductance and capacity that apply to transmission circuits are also much larger than those used here. Rough estimates using characteristic values of circuit constants make it seem probable that the net result is a delay of reignition by periods of time of the order of magnitude of those illustrated in the oscillograms.

There will of course be great differences in the rates of voltage recovery according to the location of a circuit breaker in a transmission system. There seems no doubt, for example, but that the voltage rise in a circuit breaker just off a generating station bus will take place very much more rapidly than in one located at the load end of a 100-kv. line of considerable length.

The current density in an arc is usually approximately constant. Changes in total current result in an in-

crease or decrease in the cross sectional area of the arc stream and in the area of the cathode spot from which the electron stream emerges. As yet no reason is apparent for believing that the decreasing current in an arc of 1,000 amperes r. m. s. value will cease at a minimum value any higher than the 0.05 to 0.6 ampere found here to apply in the case of a 25-ampere arc. The larger arc current will produce a higher electrode temperature, and this might easily favor a lower arc-failure current. In an application to practical problems the arc-failure current is of much less importance on high voltage than on moderate voltage circuits so far as its effect on the rate of voltage recovery is concerned.

The arc studied in this investigation was $\frac{3}{4}$ in. long. When interrupting a short circuit of any considerable magnitude a circuit breaker arc will be maintained to a considerably greater length than this, and while it lasts each half-cycle will end with a considerably greater electrode spacing than did the preceding one.

The voltage required for reignition in the case of extremely rapid voltage recovery is approximately the glow voltage. With a moderately short electrode spacing this is much higher than the normal arc voltage that exists during the greater part of each half-cycle. Most of the potential difference between electrodes in the glow occurs in the cathode drop associated with the positive ion space-charge adjacent to the cathode. This cathode drop is of course not affected by increased electrode spacing. As the electrodes separate the glow voltage increases, but not nearly in proportion to the change in spacing, as the increase takes place entirely in the region between the cathode drop and the anode.

As the cathode drop in the normal arc is a much smaller fraction of the total than is that in the glow, the arc voltage may be expected to increase much more rapidly than the glow voltage as the electrodes spread. By the time the separation is several inches the two may be approximately equal. It is very unlikely, however, that the short-time reignition voltage will ever be less than the normal arc voltage. If it were to be so, a complete half-cycle of glow, at low current and glow voltage, would immediately precede final arc extinction. Such behavior in a circuit breaker would be readily observable in the voltage and current traces of an ordinary oscillographic record, and no tests indicating it have come to the authors' attention. Low-voltage arcs between copper rods have, however, been observed to extinguish immediately following a half-cycle at glow rather than at arc voltage, the extinction resulting from gradual electrode separation.

In any case, the short-time reignition voltage will be greater for each successive half-cycle as the electrodes separate. In addition to producing this change, the greater spacing permits readier access of relatively cold oil vapors and in this and other ways accelerates the deionization that is taking place in the arc region

prior to reignition. This acceleration appears as an increase of slope in the reignition locus. Thus the reignition locus for each half-cycle should be steeper than the previous one as well as starting at a higher value.

In Fig. 17 an illustrative set of expected reignition loci for three successive half-cycles during a circuit breaker operation has been sketched qualitatively. Each one of these represents what must inevitably be true, that at every instant of time following each current zero there is some definite voltage that would cause reignition, and that this voltage increases with the passage of time. The reignition zones for a $\frac{3}{4}$ -in. arc in air, bounded by envelope curves of Figs. 15 and 16, indicate the limits within which a reignition locus representing the behavior at the end of that particular half-cycle corresponding to a $\frac{3}{4}$ -in. separation must lie. It will be observed that the reignition curves have been

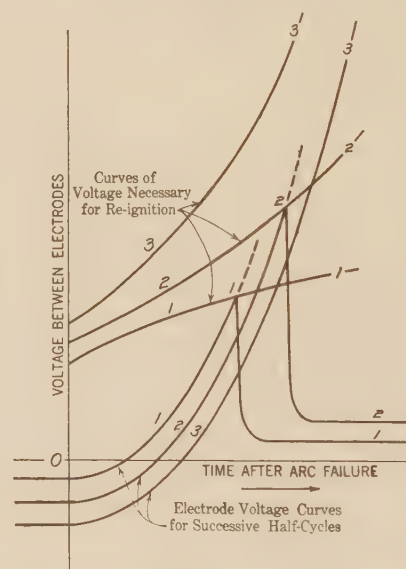


FIG. 17—REIGNITION VOLTAGE—TIME VALUES FOR CAPACITY SHUNTS WITH INCREASING ELECTRODE SPACING

represented in Fig. 17 as increasing both in short-time value and in steepness for successive half-cycles in accordance with the preceding discussion.

The lower family of curves of Fig. 17 represents typical voltage recovery curves, of the capacity shunt type, for successive half-cycles, numbered to correspond with the proper reignition locus. Each curve starts at a lower point on the graph than its predecessor because of the greater arc length and consequent greater normal arc voltage. The shapes of these curves are all alike, however, as they depend only on circuit constants external to the active breaker.

As soon as the electrode separation results in a reignition locus not intersected by the corresponding voltage rise curve, as illustrated by the No. 3 pair of Fig. 17, the circuit breaker duty is completed; the arc will not re-strike. It is evident that in any situation where the circuit constants result in a very rapid

voltage recovery, there must be compensation for this by a correspondingly rapid deionization in the region between the electrodes if the arc is not to be drawn out to an extreme length. Also, on the whole, a parallel resistance condition should be more favorable than a parallel capacity, as it would not demand ultimately so steep a slope of the reignition locus to result in a failure of the curves to intersect.

SUMMARY

A series of oscillographic records of voltage and current in an a-c. arc between copper electrodes, taken near the moment of current reversal, has indicated the following points:

1. For a brief but measurable period at the end of each half-cycle the current between the electrodes is essentially zero.

2. The values of electrode potential difference during this time can be accurately predicted if the electrical constants of the circuit and the values of arc voltage and current at the moment the arc stops are known; or conversely, if the voltage trace is known, the arc-failure current can be obtained.

3. Before current can pass in the new direction the voltage between the electrodes must rise to a definite minimum value. For copper electrodes in air at atmospheric pressure this is about 300 volts, and is very little affected by electrode shape or placement up to spacings of at least one inch.

(4) Under certain conditions the electric current flow between the electrodes just before or after the current zero, or both, may take the form recognized as a glow discharge, the glow voltage being about the same as the minimum reignition potential.

(5) If either resistance, or capacity, or both, are connected in parallel with the arc, the rate of voltage rise in the new direction is materially decreased. Associated with this time delay in the approach to reignition potential an increase in the voltage required for reignition usually occurs. The change, however, is not consistent from one half-cycle to the next, due to variable factors whose nature is not yet definitely determined.

(6) As applied to circuit breakers on power transmission and distribution systems the equations for electrode voltage during the period of zero arc current involve consideration of distributed inductance and capacity, and perhaps more important, the time of propagation of the electrical impulses between points concerned on the circuit.

(7) The rate of voltage rise toward reignition in power system circuits may be expected to be greatly affected by the position of the circuit breaker and fault relative to the inductance and distributed capacity of the system.

(8) The presence of load circuits paralleled with the arc adjacent to its input side may be expected to act in the same general manner as a resistance in parallel

with the arc, tending to cause a delay in the rise toward reignition, with a consequent lessening of the expected severity of circuit breaker duty.

(9) Increasing length of arc due to progressive electrode separation may reasonably be expected to result in a greater initial value, and greater steepness, of the reignition locus, and in a simultaneous lowering of the voltage available for reignition at any specified moment following the current zero. The arc extinguishes when at the end of some half cycle the voltage curve fails to cross the reignition locus.

ACKNOWLEDGMENT

The authors wish to acknowledge the kindness of Dr. J. Slepian for suggesting the desirability of this study, and the loan by the Detroit Edison Company of a cathode-ray oscillograph with which many of the oscillograms were taken.

Appendix A

(See Fig. 1)

Arc Voltage During Arc Current Zero. (No Arc Shunt)

$$E = e_a + L \frac{d i_L}{d t} = e_a + \frac{q_c}{C_L}$$

$$i_L + i_c = 0$$

$$\frac{d i_L}{d t} = - \frac{d i_c}{d t}$$

$$\frac{q_c}{C_L} + L \frac{d^2 q_c}{d t^2} = 0$$

$$q_c = A \cos \omega t + B \sin \omega t$$

$$\omega = \frac{1}{\sqrt{L C_L}}$$

$$e_a = E - \frac{1}{C_L} (A \cos \omega t + B \sin \omega t)$$

$$i_L = - i_c = - \frac{d q_c}{d t} = A \omega \sin \omega t - B \omega \cos \omega t$$

Initial Conditions:

$$\text{At } t = 0 \quad e_a = E_1 \text{ and } i_L = I_1$$

$$E_1 = E - \frac{1}{C_L} A \quad A = C_L (E - E_1)$$

$$I_1 = - B \omega \quad B = - \frac{I_1}{\omega}$$

$$\text{Hence } e_a = E - (E - E_1) \cos \omega t + \frac{I_1}{\omega C_L} \sin \omega t$$

$$e_a = E - E_m \cos (\omega t + \theta)$$

$$\text{where } \omega = \frac{1}{\sqrt{L C_L}}$$

E = the maximum of the supply voltage.

E_1 = the arc voltage when the arc fails.

I_1 = the arc current when the arc fails.

e_a = the arc voltage during the period $t = 0$ to $t = t_1$
(Fig. 2)

$$\theta = \tan^{-1} \frac{I_1 / \omega C_L}{E - E_1}$$

$$E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C_L} \right)^2} \quad I_1 = \sqrt{E_m^2 - (E - E_1)^2 \omega C_L}$$

$$\left(\frac{d e_a}{d t} \right)_{t=0} = \frac{I_1}{C_L}$$

Appendix B

(See Fig. 7A)

$$E = L \frac{d i_L}{d t} + R i = \frac{q_c}{C_L} + R i$$

$$i = i_L + \frac{d q_c}{d t} = \text{line current}$$

$$\frac{d i}{d t} = \frac{d i_L}{d t} + \frac{d^2 q_c}{d t^2}$$

$$E = \frac{q_c}{C_L} + R \left(i_L + \frac{d q_c}{d t} \right)$$

$$0 = \frac{1}{C_L} \frac{d q_c}{d t} + R \left(\frac{d i_L}{d t} + \frac{d^2 q_c}{d t^2} \right)$$

$$0 = q_c + \frac{L}{R} \frac{d q_c}{d t} + L C_L \frac{d^2 q_c}{d t^2}$$

If $q_c = e^{mt}$

$$1 + \frac{L}{R} m + L C_L m^2 = 0$$

$$m = -\frac{1}{2 R C_L} \pm \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}$$

$$\text{I. If } \frac{1}{4 R^2 C_L^2} < \frac{1}{L C_L} R > R_c \quad R_c = \frac{1}{2} \sqrt{\frac{L}{C_L}}$$

$$q_c = e^{-\frac{1}{2 R C_L} t} [A \cos \omega t + B \sin \omega t]$$

$$\text{where } \omega = \sqrt{\frac{1}{L C_L} - \frac{1}{4 R^2 C_L^2}}$$

$$e_a = E - \frac{q_c}{C_L} = E - \frac{1}{C_L} e^{-\frac{1}{2 R C_L} t} [A \cos \omega t + B \sin \omega t]$$

$$i_c = \frac{d q_c}{d t} = -C_L \frac{d e_a}{d t}$$

$$= e^{-\frac{1}{2 R C_L} t} [-A \omega \sin \omega t + B \omega \cos \omega t] - \frac{1}{2 R C_L} e^{-\frac{1}{2 R C_L} t} [A \cos \omega t + B \sin \omega t]$$

Initial Conditions:

$$\text{At } t = 0 \quad e_a = E_1 \text{ and } i_c = -I_1$$

$$E_1 = E - \frac{1}{C_L} A \quad A = C_L (E - E_1)$$

$$-I_1 = B \omega - \frac{1}{2 R C_L} A = B \omega - \frac{E - E_1}{2 R}$$

$$B = \frac{1}{\omega} \left(-I_1 + \frac{E - E_1}{2 R} \right)$$

Hence

$$e_a = E - e^{-\frac{1}{2 R C_L} t} \left[(E - E_1) \cos \omega t + \frac{1}{\omega C_L} \left(-I_1 + \frac{E - E_1}{2 R} \right) \sin \omega t \right]$$

This reduces to the value given for e_a in Appendix A when $R = \infty$. I_1 = the arc current at the breaking point.

$$\left(\frac{d e_a}{d t} \right)_{t=0} = \frac{I_1}{C_L}$$

$$\text{II. If } \frac{1}{4 R^2 C_L^2} > \frac{1}{L C_L} \quad R < R_c \quad R_c = \frac{1}{2} \sqrt{\frac{L}{C_L}}$$

$$\text{Let } \beta = \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}$$

$$q_c = e^{-\frac{1}{2 R C_L} t} [A \cosh \beta t + B \sinh \beta t]$$

$$e_a = E - \frac{q_c}{C_L}$$

$$= E - \frac{1}{C_L} e^{-\frac{1}{2 R C_L} t} [A \cosh \beta t + B \sinh \beta t]$$

$$i_c = \frac{d q_c}{d t} = -C_L \frac{d e_a}{d t}$$

$$= e^{-\frac{1}{2 R C_L} t} [A \beta \sinh \beta t + B \beta \cosh \beta t]$$

$$- \frac{1}{2 R C_L} e^{-\frac{1}{2 R C_L} t} [A \cosh \beta t + B \sinh \beta t]$$

Initial Conditions:

$$\text{At } t = 0 \quad e_a = E_1 \text{ and } i_c = -I_1$$

$$E_1 = E - \frac{1}{C_L} A \quad A = C_L (E - E_1)$$

$$-I_1 = B \beta - \frac{1}{2 R C_L} C_L (E - E_1)$$

$$B = \frac{1}{\sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}} \left[-I_1 + \frac{E - E_1}{2 R} \right]$$

Hence

$$e_a = E - e^{-\frac{1}{2RC_L} t} \left[(E - E_1) \cosh \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}} t + \right. \\ \left. + \frac{-I_1 + \frac{E - E_1}{2R}}{C_L \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}} \sinh \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}} t \right] \\ \left(\frac{d e_a}{d t} \right)_{t=0} = \frac{I_1}{C_L}$$

Appendix C

(See Fig. 7B)

$$E = L \frac{d i}{d t} + R i + \frac{q}{C}$$

$$E = L \frac{d^2 q}{d t^2} + R \frac{d q}{d t} + \frac{q}{C}$$

$$O = L \frac{d^3 q}{d t^3} + R \frac{d^2 q}{d t^2} + \frac{1}{C} \frac{d q}{d t}$$

If

$$q = e^{mt}$$

$$O = L m^3 + R m^2 + \frac{1}{C} m$$

$$m = 0$$

$$O = L m^2 + R m + \frac{1}{C}$$

$$m = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$\text{I. If } \frac{R^2}{4L^2} < \frac{1}{LC}$$

$$q = C E + e^{-\frac{R}{2L} t} [A \cos \omega t + B \sin \omega t]$$

where

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$i = \frac{d q}{d t} = e^{-\frac{R}{2L} t} \left[\left(\omega B - \frac{R}{2L} A \right) \cos \omega t \right. \\ \left. - \left(\omega A + \frac{R}{2L} B \right) \sin \omega t \right]$$

$$e_a = \frac{q}{C} = E + \frac{1}{C} e^{-\frac{R}{2L} t} [A \cos \omega t + B \sin \omega t]$$

Initial Conditions:

$$\text{At } t = 0 \quad e_a = E_1, \quad i = I_1 \text{ and } q = C E_1$$

$$C E_1 = C E + A \quad A = -C (E - E_1)$$

$$I_1 = \omega B + \frac{R}{2L} C (E - E_1)$$

$$B = \frac{1}{\omega} \left[I_1 - \frac{R}{2L} C (E - E_1) \right]$$

Hence

$$e_a = E + e^{-\frac{R}{2L} t} \left[- (E - E_1) \cos \omega t \right. \\ \left. + \frac{1}{\omega C} \left(I_1 - \frac{R}{2L} C (E - E_1) \right) \sin \omega t \right] \\ \left(\frac{d e_a}{d t} \right)_{t=0} = \frac{I_1}{C}$$

II. If $R = 0$,

$$e_a = E - \left[(E - E_1) \cos \omega t - \frac{I_1}{\omega C} \sin \omega t \right]$$

$$i = I_1 \cos \omega t + \omega C (E - E_1) \sin \omega t$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$e_a = E - E_m \cos (\omega t + \theta)$$

$$\text{where } E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C} \right)^2}$$

$$\tan \theta = \frac{I_1}{\omega C (E - E_1)}$$

$$I_1 = \omega C \sqrt{E_m^2 - (E - E_1)^2}$$

which is the same as the case of Appendix A.

$$\left(\frac{d e_a}{d t} \right)_{t=0} = \frac{I_1}{C}$$

